Supplementary appendix to 'Search intensity, wage segmentation and the wage distribution'

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1 Proof of Proposition 5

We have

$$\frac{G''(w)}{G'(w)} = \frac{\chi(w)}{\alpha(\theta(w))(y-w)(w-R)} \tag{1}$$

Using the definitions of the functions $\chi(w)$ given in the proof of Proposition 4, equation (1) becomes:

$$\frac{G''(w)}{G'(w)} = \frac{\gamma(y-R)}{(y-w)(w-R)} - \frac{1+\gamma-\alpha}{\alpha(y-w)} \tag{2}$$

Integrating this equation with the condition $\int_R^y G'(w)dw = 1$ yields

$$G'(w) = \frac{(y-w)^{\frac{(1+\gamma)(1-\alpha)}{\alpha}}(w-R)^{\gamma}}{\int_R^y (y-\xi)^{\frac{(1+\gamma)(1-\alpha)}{\alpha}}(\xi-R)^{\gamma}d\xi} \tag{3}$$

The cdf of the actual normalized wage is such that

$$H_G(\omega) = \Pr\left(\frac{w-R}{y-R} \leq \omega\right) = \Pr\left(w \leq R + \omega(y-R)\right) = G\left[R + \omega(y-R)\right] \tag{4}$$

Therefore one gets $H_G'(\omega) = (y-R)G'\left[R+\omega(y-R)\right]$, and the result follows.

2 Proof that there are no multiple offers

We closely follow Mortensen (1986) who shows in the standard job-search model that the probability of receiving more than one offer conditional on the fact that the worker receives at least one offer tends to 0 as the time interval tends to 0. The proof must be adapted to account for the fact that there are a continuum of markets in our model. Mortensen discretizes time, and interprets continuous time as a case in which the time interval between two dates tends to 0. We not only discretize time but also space: the wage distribution is cut into intervals of equal distance. The case of a continuum of markets corresponds to the case where such distance tends 0.

Consider the function $\lambda(w) = x\left[s(w)\right]\theta(w)\, m\left[\theta(w)\right]$ defined over $[R, y]$. Our proof does not depend on the particular form taken by the function $\lambda(w)$ in our model. It is valid for any function $\lambda(w)$ that is positive and continuous on the interval $[R, y]$. Cut the interval $[R, y]$ into

n intervals of the same length $dw = (y - R)/n$. On interval $i \in \{1, ..., n\}$, there is a unique wage $w_i = R + (i-1)\,dw$. Now, consider interval i, and cut it into m intervals. Assume that the probability of receiving an offer from any such interval is $\lambda_i dwdt/m$ over the period dt, with $\lambda_i = \lambda(w_i)$.

Let X_i be the number of offers received from interval i over the period dt. The probability of receiving $k_i \in \{0, ..., m\}$ offers is:

$$\Pr(X = k_i) = C_m^{k_i} (\lambda_i dwdt/m)^{k_i} (1 - \lambda_i dwdt/m)^{m-k_i} \tag{5}$$

As $m \to \infty$, it tends to

$$\Pr(X = k_i) = e^{-\lambda_i dwdt} \frac{(\lambda_i dwdt)^{k_i}}{k_i!} \tag{6}$$

Hence, X_i follows the Poisson law of parameter $\lambda_i dwdt$.

Now, consider the random variable $X = \sum\limits_{i=1}^{n} X_i$ which is the total number of offers received from all the intervals over the period dt. As the different variables are independent draws from Poisson laws, the sum of the draws also follows a Poisson law, whose parameter is the sum of the parameters of the different Poisson laws. Hence,

$$\Pr(X = k) = \Pr\left(\sum_{i=1}^{n} X_i = k\right) = e^{-\lambda dt} \frac{(\lambda dt)^k}{k!}, \quad \text{with } \lambda = \sum_{i=1}^{n} \lambda_i dw \tag{7}$$

As $n \to +\infty$, we obtain that X follows the Poisson law of parameter $\lambda = \lim\limits_{n \to +\infty} \sum\limits_{i=1}^{n} \lambda_i dw = \int_R^y \lambda(w) dw$. The remainder of the proof is standard. Following Mortensen (1986),

$$\frac{\Pr(X = k)}{dt} = e^{-\lambda dt} \frac{\lambda^k (dt)^{k-1}}{k!} \tag{8}$$

The right-hand side tends to 0 when dt tends to 0 for all $k > 1$. Similarly, it tends to λ when dt tends to 0 when $k = 1$. It follows that the probability of receiving more than one offer conditional to the fact that the worker receives at least one offer tends to 0 when dt tends to 0.